

```
$ python pi_darts.py
```

Guessing π with Random Darts

```
// using randomness to find one of the most famous numbers in maths
```

— Dr. Hughes

A Question

How could we find out the value of π , without knowing π ?

Today: a dartboard, some random numbers, and Python.

What Is π ?

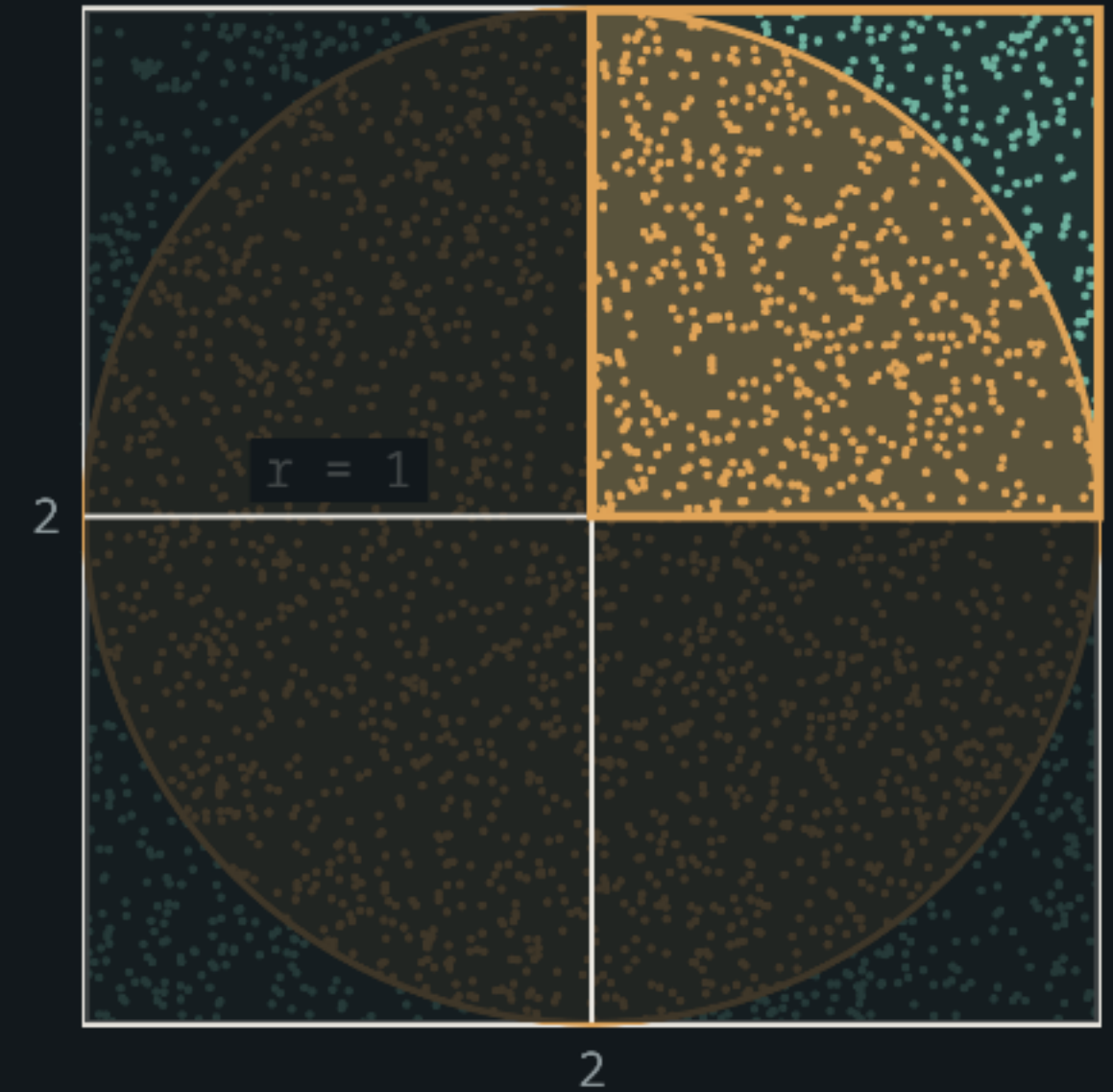
π is in every circle's area: $\pi \times r^2$

circle, $r = 1$ $\pi \times 1^2 = \pi$

square around it $2 \times 2 = 4$

darts land evenly, so $\frac{\text{inside}}{\text{total}} \approx \frac{\pi}{4}$

Every quarter is the same, so we only need one: our dartboard.



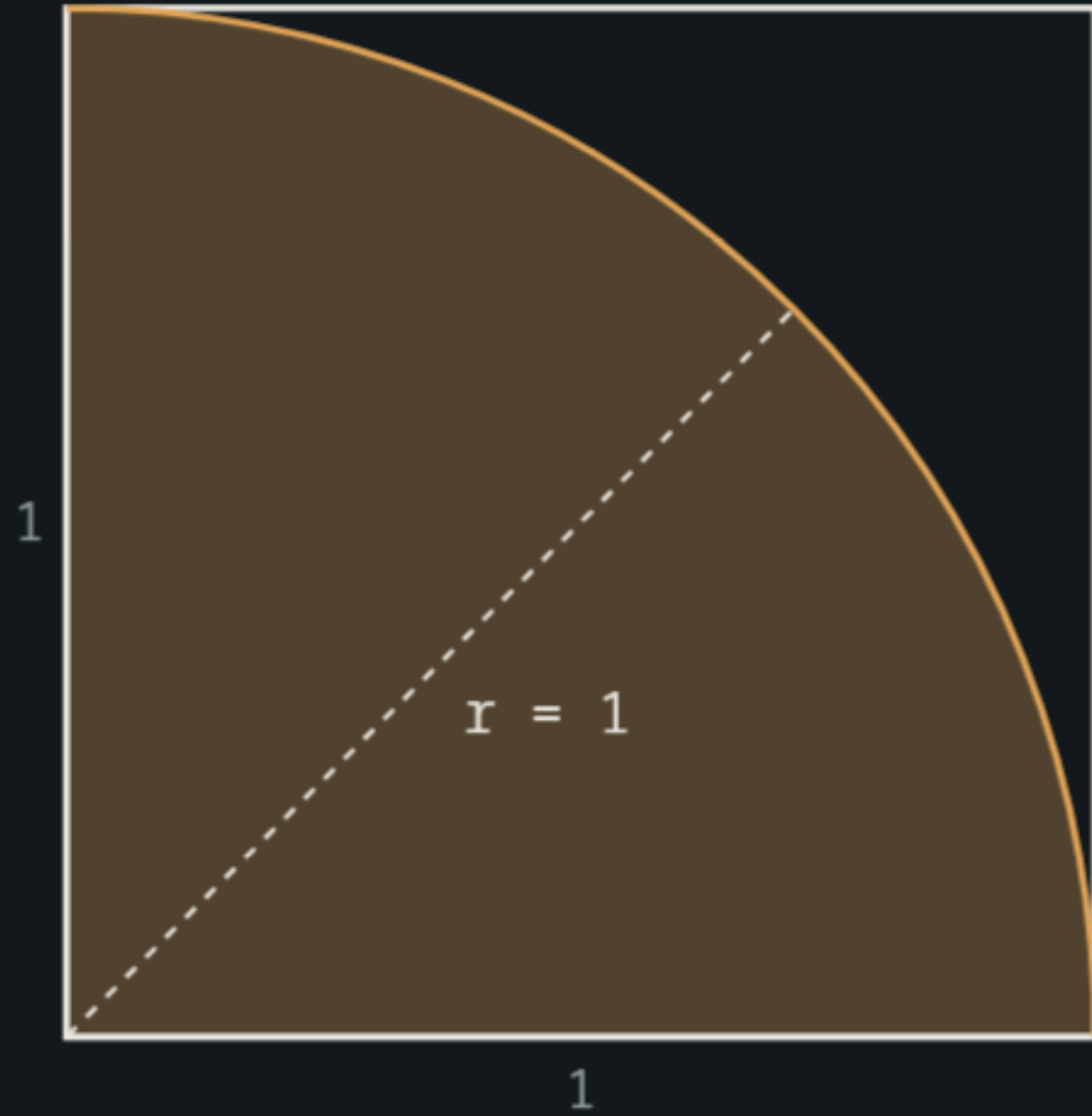
0.7917

inside $\div$ total

0.7854

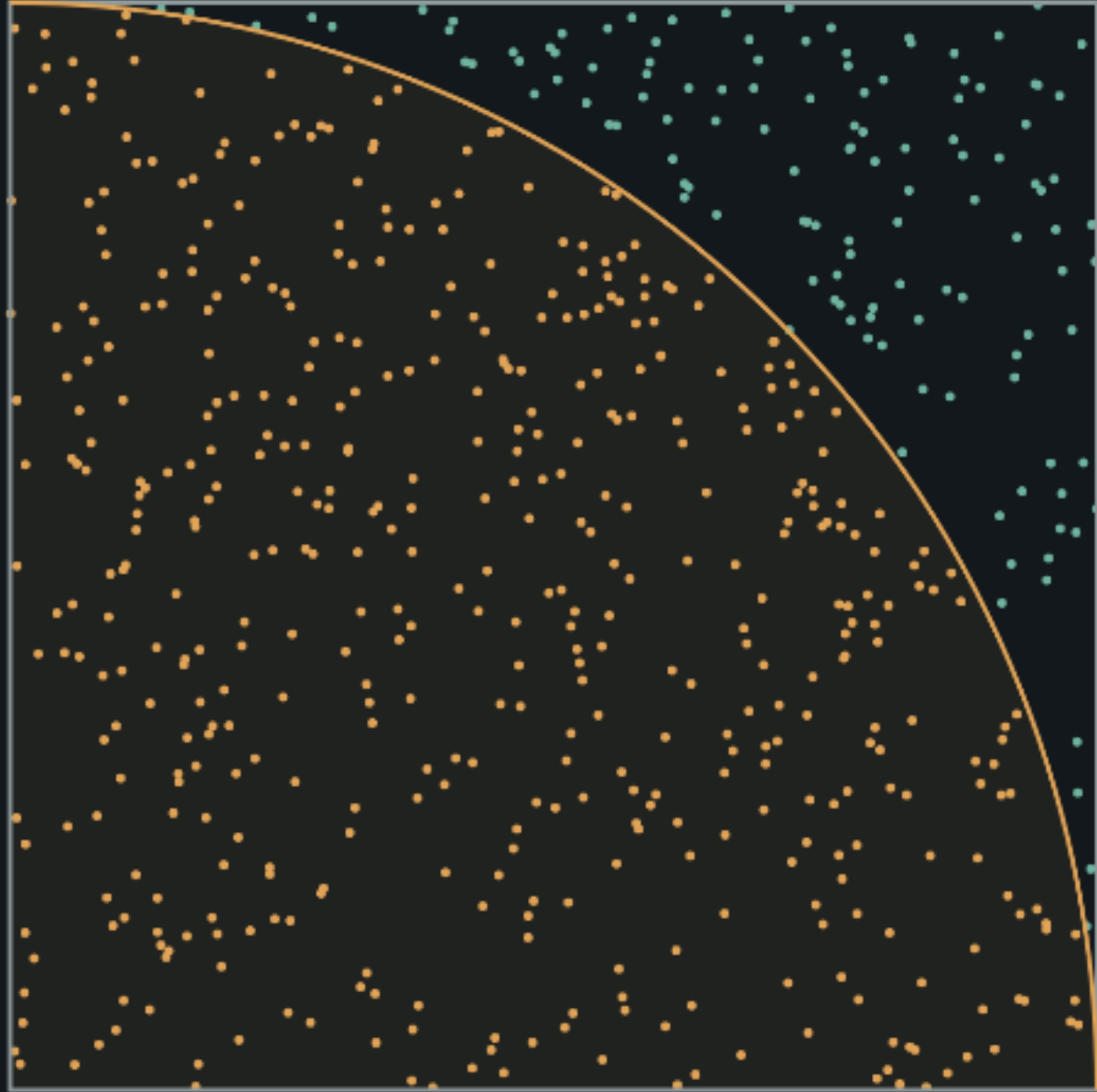
$\pi \div 4$

The Dartboard



One quarter: a square, **1** wide and **1** tall.
Inside it, a quarter circle with radius $r = \mathbf{1}$.

Throw Random Darts



Every dart lands at a **random** spot in the square.

Some land inside the quarter circle, some outside.

Can Python throw the darts for us?

474	126	600
● inside	● outside	total

Random Numbers in Python

```
>>> import random  
  
>>> random.random()  
0.7215423190351417  
  
>>> random.random()  
0.0931705372244931
```

// a new random number between 0 and 1, every time

Every Number Equally Likely

`random.random()` is **uniform**: every value from 0 to 1 is equally likely.



20,000

random numbers so far

// this matters: uniform x and y means darts land evenly, so the count matches the area

Throw One Dart

```
1 import random

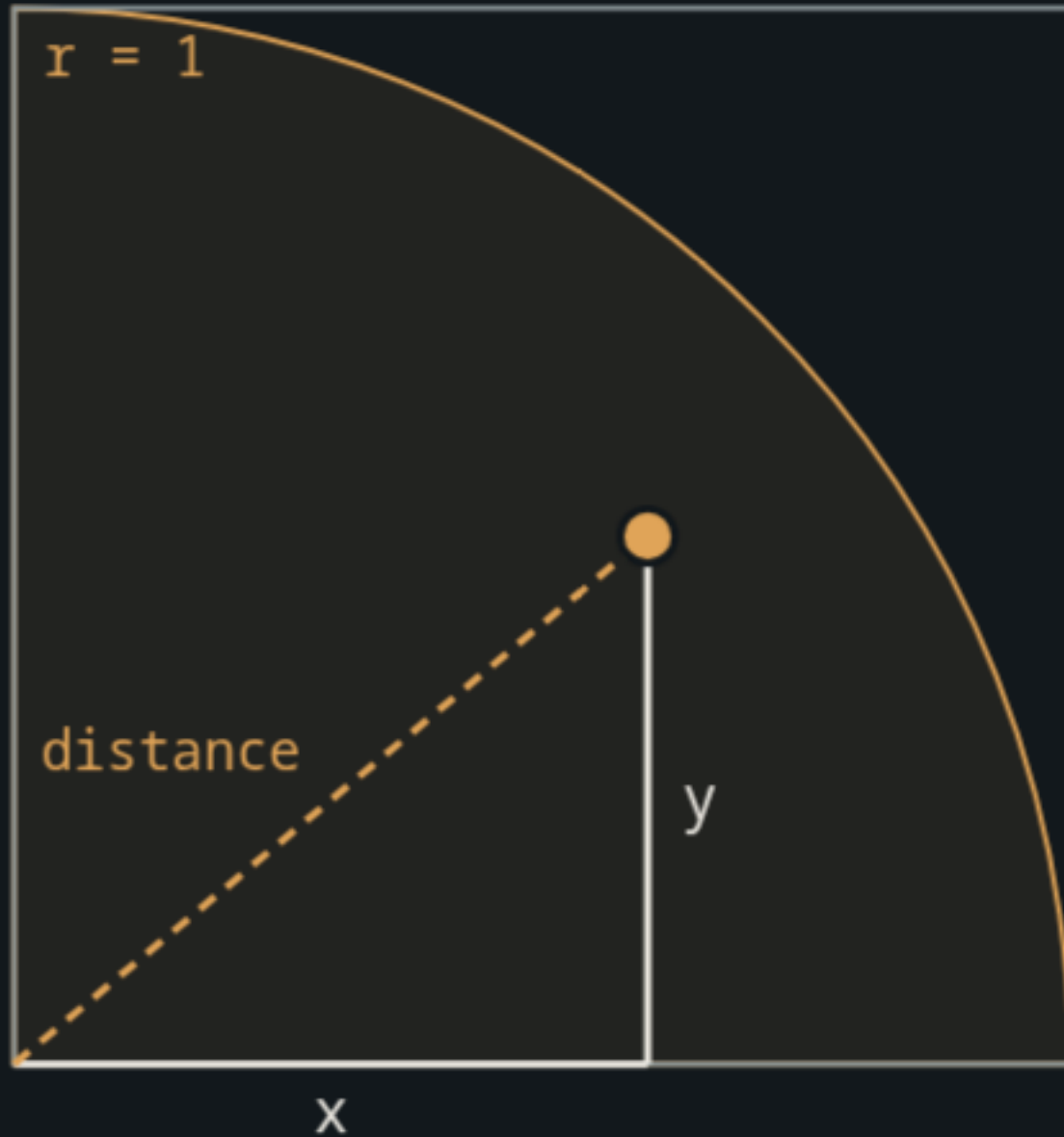
3 x = random.random()  # how far across
4 y = random.random()  # how far up

6 print(x, y)
```

```
0.6012847361 0.4987213094
```

// two random numbers = one spot in the square

Is the Dart Inside?



across

$$x = 0.60$$

up

$$y = 0.50$$

Euclidean distance

$$\text{distance} = \sqrt{x^2 + y^2}$$

from the corner

$$\text{distance} = 0.78$$

$$\text{distance} \leq r \rightarrow \text{inside} \checkmark$$

// inside = distance from the corner $\leq r$

Why $x*x + y*y$?

Euclidean distance $\text{distance} = \sqrt{x^2 + y^2}$

inside means $\text{distance} \leq r$

square both sides $x^2 + y^2 \leq r^2$

$r = 1$, and $1^2 = 1$ $x^2 + y^2 \leq 1$

in Python $x*x + y*y \leq 1$

Inside or Outside?

```
1 import random

3 x = random.random()
4 y = random.random()

6 if x*x + y*y <= 1:
7     print("inside!")
8 else:
9     print("outside")
```

Your Turn: 5 Minutes

» Type this in and run it a few times.

```
1 import random

3 x = random.random()
4 y = random.random()

6 if x*x + y*y <= 1:
7     print("inside!")
8 else:
9     print("outside")
```

Finished? Use a `for` loop to throw 10 darts.

5:00

start

reset

Throw 10,000 Darts

```
1 import random

3 total = 10000
4 inside = 0

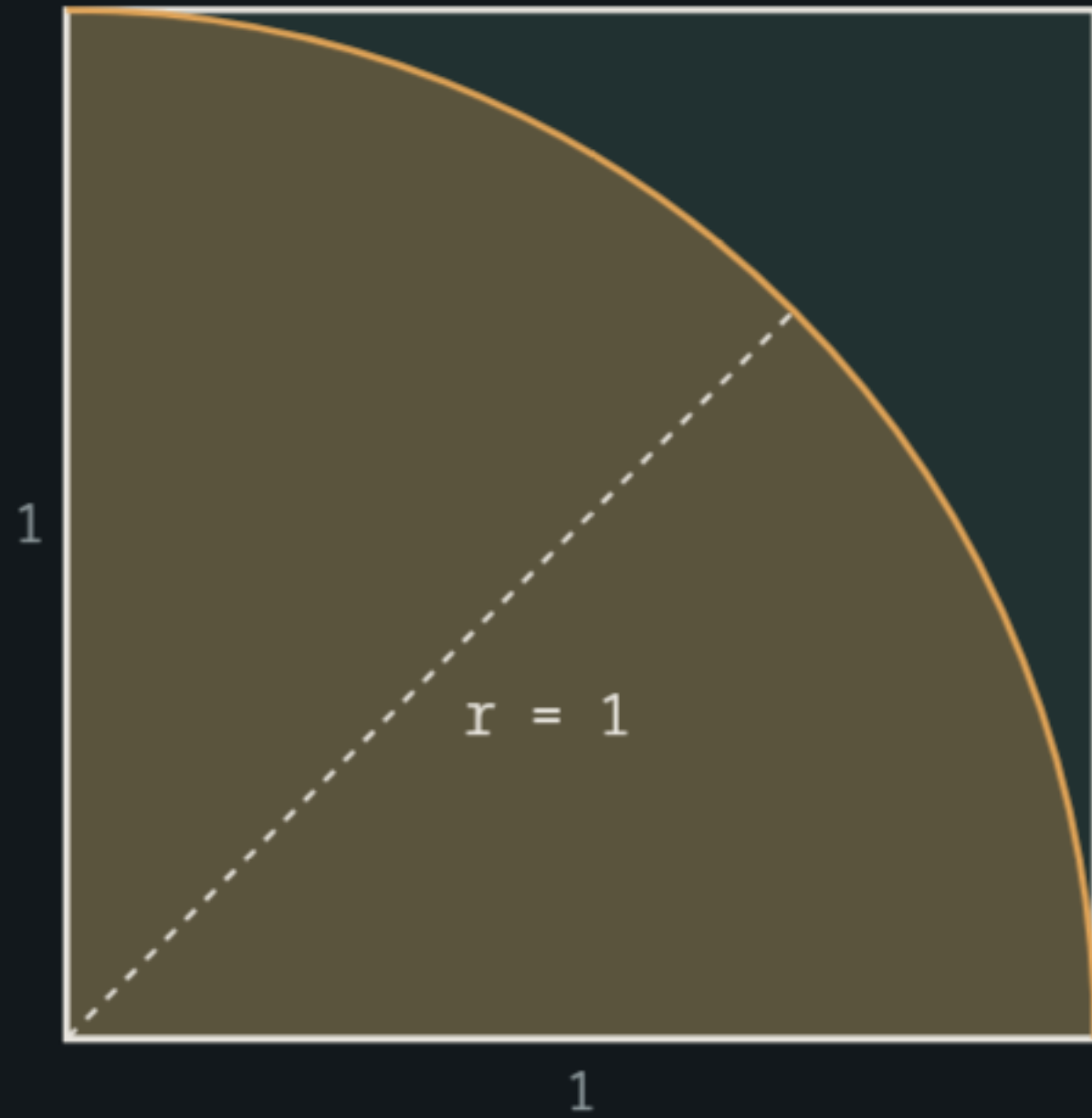
6 for i in range(total):
7     x = random.random()
8     y = random.random()
9     if x*x + y*y <= 1:
10        inside = inside + 1

12 print(inside)
```

7853

// $7,853 \div 10,000 \approx 0.785$. Remember $\pi \div 4$?

Why 0.785? Compare the Areas



square

$$1 \times 1 = 1$$

quarter circle

$$\frac{1}{4} \times \pi \times 1^2 = \frac{\pi}{4}$$

$$\frac{\text{quarter circle}}{\text{square}} = \frac{\pi}{4} = 0.785\dots$$

The Trick

$$\frac{\text{inside}}{\text{total}} \approx \frac{\text{quarter circle}}{\text{square}} = \frac{\pi}{4}$$

So multiply both sides by 4...

$$\pi \approx 4 \times \frac{\text{inside}}{\text{total}}$$

One More Line

```
1 import random

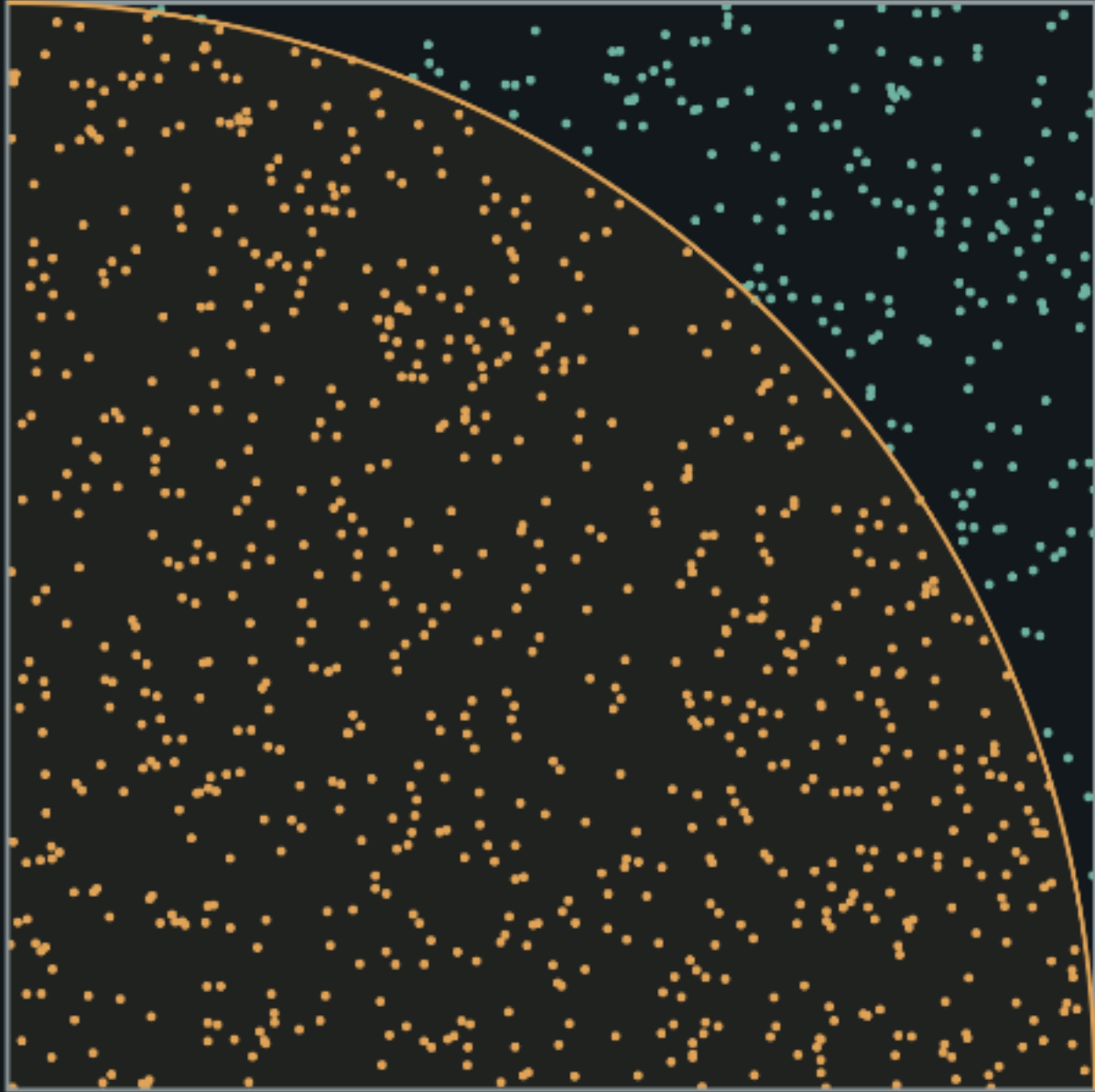
3 total = 10000
4 inside = 0

6 for i in range(total):
7     x = random.random()
8     y = random.random()
9     if x*x + y*y <= 1:
10        inside = inside + 1

12 pi_guess = 4 * inside / total
13 print(pi_guess)
```

3.1412

Try It Live



$\pi \approx 3.2267$ off by 0.0851

797 191 988

● inside ● outside total

+10

+100

+1,000

+10,000

reset

More Darts, Better Guess



throw a new set

after 98,447 darts: $\pi \approx 3.1402$

This Has a Name

A **Monte Carlo method** uses lots of random tries to answer a question.

Named after a famous casino in Monaco 🎲

Used for weather forecasts, video game AI, and physics simulations.

Take It Further

» Run it with **100**, **10,000** and **1,000,000** darts. How close do you get?

Extra: print the guess every 1,000 darts, so you can watch it settle.

What You Learned Today

- ✓ `random.random()` is uniform, so darts land evenly
- ✓ Euclidean distance from the corner: $\sqrt{x^2 + y^2}$
- ✓ `inside = distance ≤ r`, so $x*x + y*y \leq 1$
- ✓ the quarter circle is $\pi \div 4$ of the square
- ✓ $\pi \approx 4 \times \text{inside} \div \text{total}$

Get the Slides



Scan to find these slides, and the others from class.

`aoifehughes.github.io/teaching/slides/`